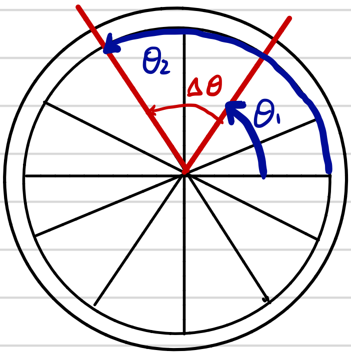
Rotation of Rigid Bodies

Basic Equations and Concepts!

(I) Rotational Kinematics:

Same equations we used before but with $x \mapsto \theta$, $v \mapsto \omega$, and $a \mapsto \alpha$:



$$\boxed{\omega_z = \frac{d\theta}{dt}, \qquad \alpha_z = \frac{d\omega_z}{dt} = \frac{d^2\theta}{dt}}$$

For constant α_z:

$$\theta = \theta_0 + \omega_{iz} t + \frac{1}{2}\alpha_z t^2$$

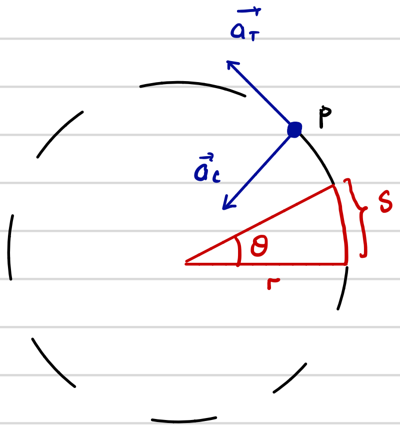
$$\omega_{fz} = \omega_{iz} + \alpha_z t$$

$$\omega_{fz}^2 = \omega_{iz}^2 + 2\alpha_z \Delta\theta$$

(II) Relating linear and angular kinematics:

$\vec{a_T}$ =: tangential acceleration of P

$\vec{a_c}$ =: centripetal acceleration of P



From precalculus, you know:

$$\boxed{S = r\theta} \Rightarrow \frac{ds}{dt} = r\frac{d\theta}{dt}$$

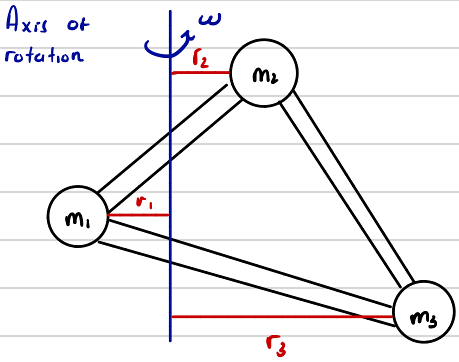
$$\boxed{v = r\,\omega}$$

$$\Rightarrow \underbrace{\frac{dv}{dt}}_{a_T} = r\underbrace{\frac{d\omega}{dt}}_{\alpha}$$

$$\boxed{a_T = r\,\alpha}$$

Also, you have learned that $\boxed{a_c = \frac{v^2}{r}} = \frac{(r\omega)^2}{r} \boxed{= \omega^2 r}$

(III) Moment of inertia (I) and rotational kinetic energy (K):

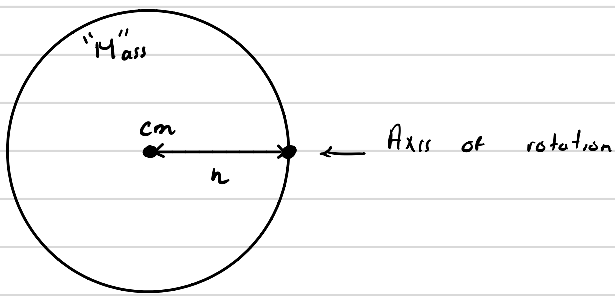


$$\boxed{I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + \cdots}$$

$$\boxed{K = \frac{1}{2} I \omega^2}$$

(IV) Parallel axis theorem:

$$\boxed{I = I_{cm} + M h^2}$$



Problem Solving

I. (P. 9.10 a)) $\omega_f = 200 \frac{\cancel{rev}}{\cancel{min}} \left(\frac{1 \cancel{min}}{60\ s}\right)\left(\frac{2\pi\ rad}{1\ \cancel{rev}}\right) = 20.94\ rad/s$

$\omega_i = 500 \frac{rev}{min} = 53.36\ rad/s$

$t = 4.00\ s$

a) $\omega_f = \omega_i + \alpha t \Rightarrow \boxed{\alpha =} \frac{\omega_f - \omega_i}{t} = \frac{20.94\ rad/s - 53.36\ rad/s}{4.00\ s}$

$\boxed{\approx -8.105\ rad/s^2}$

$\boxed{\Delta\theta =} \omega_i t + \frac{1}{2}\alpha t^2 = (53.36\ rad/s)(4.00\ s) + \frac{1}{2}(-8.105\ rad/s^2)(4.00\ s)^2$

$\approx 28.6\ rad \boxed{= 23.6\ rev}$

II. (P. 9.17) $\cancel{\omega_f}^{2\,0} = \omega_1^2 + 2\alpha\Delta\theta_1 \Rightarrow \omega_1^2 = -2\alpha\,\Delta\theta_1$

Similarly, $\omega_3^2 = -2\alpha\,\Delta\theta_3$ and $\omega_3 = 3\omega_1$ then

$\omega_3^2 = 9\omega_1^2 = 9(-2\alpha\,\Delta\theta_1) = -2\alpha\,\Delta\theta_3$

$\Rightarrow \boxed{\Delta\theta_3 =} 9\Delta\theta_1 \boxed{= 9 \text{ revolutions}}$

III. (P. 9.18) a) $V = 25\ cm/s = 0.25\ m/s$; $r = 1.25\ m$

$\omega = ?$ $\Rightarrow \boxed{\omega =} \frac{V}{r} = \frac{0.25\ m/s}{1.25\ m} = \frac{1}{7} \frac{rad}{s} \boxed{= 1.9 \frac{rev}{min}}$

b) $a_T = \frac{1}{8} g$; $r = 1.25\ m$

$\alpha = \frac{a_T}{r} = \frac{\frac{1}{8}(9.8\ m/s^2)}{1.25\ m} = 0.98\ rad/s^2$

c) $S = 3.25\ m \quad ; \quad r = 1.25\ m$

$$\boxed{\theta =} \frac{s}{r} = \frac{3.25\ m}{1.25\ m} \boxed{= 2.6\ rad}$$

in degrees $\quad 2.6\ rad \left(\frac{360^\circ}{2\pi\ rad} \right) = 149^\circ$

IV. (P. 9.21 a)) $\quad r = 20.0\ cm = 0.2\ m \quad ; \quad \omega_i = 0 \quad ; \quad \alpha = 3.00\ rad/s^2$

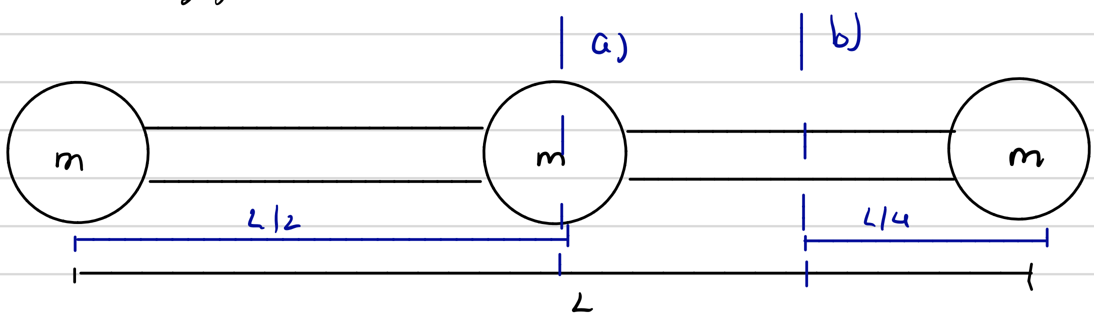
$\Delta\theta = 4\pi\ rad.$

a) $a_{rad} = \omega^2 r \quad , \quad \omega^2 = \cancel{\omega_i^2}^{\,0} + 2\alpha\Delta\theta$

$$\Rightarrow \boxed{a_{rad}} = 2\alpha\Delta\theta r = 2\,(3.00\ rad/s^2)(4\pi\ rad)(0.2\ m)$$

$$\boxed{\approx 15.07\ m/s^2}$$

V. (P. 9.32) $\quad$ Negligible mass means no moment of inertia of the rod.



a) $\boxed{I =} \sum_i m_i r_i^2 = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = 2m\frac{L^2}{4} \boxed{= \frac{1}{2} m L^2}$

b) $\boxed{I =} \sum_i m_i r_i^2 = 2m\left(\frac{L}{4}\right)^2 + m\left(\frac{3L}{4}\right)^2 = 2m\frac{L^2}{16} + \frac{9mL^2}{16}$

$$\boxed{= \frac{11}{16} m L^2}$$

VI. (P. 9.47) $M_p = 2.50\ kg$; $r = 20.0\ cm = 0.2\ m$; $m = 1.5\ kg$

$K = 4.5\ J$

?

$$K = \frac{1}{2} I \omega^2 \quad \text{where} \quad I = \frac{1}{2} M_p r^2$$

?

$$\omega^2 = \cancel{\omega_i^2}^{\,0} + 2\alpha\Delta\theta = 2\frac{a_T}{r}\cdot\frac{x}{r} \Rightarrow \omega^2 r^2 = 2 a_T x$$

From Newton's Laws : $mg - T = ma_T$, $Tr = I\alpha = I\frac{a_T}{r}$

$$\Rightarrow Tr = \frac{1}{2} M_p r^2 \frac{a_T}{r} \Rightarrow T = \frac{1}{2} M_p a_T$$

$$\Rightarrow mg - \frac{1}{2} M_p a_T = m a_T \Rightarrow a_T = \frac{mg}{\frac{1}{2}M_p + m}$$

$$\therefore K = \frac{1}{2}\cdot\frac{1}{2} M_p r^2 \omega^2 = \frac{1}{4} M_p (2 a_T x)$$

$$= \frac{1}{2} M_p \frac{mg}{\frac{1}{2}M_p + m} x = \frac{M_p mg}{M_p + 2m} x$$

$$\Rightarrow \boxed{x =} \frac{M_p + 2m}{M_p mg} K \boxed{= 0.673\ m}$$

VII. (P. 9.54) $I_{cm} = MR^2$ $d = R$

$$\boxed{I =} MR^2 + MR^2 = \boxed{2MR^2}$$