Problemas Principales : Mecánica Cuántica

Preguntas conceptuales :

1. F
2. d
3. b
4. c, d

Problemas :

P1.

$$|\Psi(x)|^2 = \begin{cases} 0.111x + 0.333 & -3 \le x \le 0 \\ -0.111x + 0.333 & 0 \le x \le 3 \end{cases}$$

$$P(-0.05 \le x \le 0.05) = \int_{-0.05}^{0} (0.111x + 0.333)\,dx$$

$$+ \int_{0}^{0.05} (-0.111x + 0.333)\,dx = 2\int_{0}^{0.05} (-0.111x + 0.333)\,dx$$

$$\approx 0.033$$

$$N_e = 4100 \cdot 0.033 \approx 136 \text{ electrones}$$

P2.

$$1 = \int_{-\infty}^{\infty} |\Psi(x)|^2\,dx = \int_{0}^{\infty} b^2 e^{-\frac{2x}{6.4}}\,dx$$

$$\left( \int e^{\alpha x}\,dx = \frac{e^{\alpha x}}{\alpha} \right) \quad = b^2 \frac{e^{-\frac{x}{3.2}}}{-\frac{1}{3.2}} \Bigg|_0^{\infty}$$

$$= b^2 (3.2) \Rightarrow b = \sqrt{\frac{1}{3.2}} = 0.56 \text{ nm}^{-1/2}$$

P3. Igual que P2:

$$1 = \int_0^\infty A^2 e^{-\frac{2x}{2.3}} dx = A^2 \frac{e^{-\frac{x}{1.15}}}{-\frac{1}{1.15}} \Big|_0^\infty$$

$$= A^2 (1.15) \Rightarrow A = \sqrt{\frac{1}{1.15}} = 0.93 \ nm^{-1/2}$$

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$$P(1.15\ nm < x < 1.84\ nm) = \int_{1.15}^{1.84} (0.93)^2 e^{-\frac{x}{1.15}} dx$$

$$\approx 0.165$$

P4. $$1 = \int_{-L}^{L} A^2 dx = A^2 (2L) \Rightarrow A = \sqrt{\frac{1}{2L}}$$

P5. $$E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2};$$

$$E_{min} = E_1 = \frac{\pi^2 \hbar^2}{2 m_e a^2} \approx \frac{(3.14)^2 (1.05 \times 10^{-34} J \cdot s)^2}{2 (9.11 \times 10^{-31} kg) (8.9 \times 10^{-9} m)^2}$$

$$\approx 7.53 \times 10^{-22} J$$

$$\approx 0.0047 \ eV$$

P6. $$E_n = \frac{n^2 \pi^2 \hbar^2}{2 m_e a^2} > 100 \ eV \left( \frac{1.6 \times 10^{-19} J}{1 \ eV} \right)$$

$$\Rightarrow n > \sqrt{(1.6 \times 10^{-17} J) \frac{2 m_e a^2}{\pi^2 \hbar^2}}$$

$$\Rightarrow \quad n = \sqrt{\frac{(1.6 \times 10^{-19}\,J) \cdot 2 \cdot 9.11 \times 10^{-31}\,kg \cdot (2.6 \times 10^{-9}\,m)^2}{\pi^2 \cdot (1.05 \times 10^{-34}\,J \cdot s)^2}} \approx 42.5$$

$$\therefore \quad n = 43$$

P7. $\quad E_1 = \frac{\pi^2 \hbar^2}{2ma^2} \qquad E_n = n^2 \frac{\pi^2 \hbar^2}{2ma^2} \Rightarrow E_n = n^2 E_1$

$$E_2 = 4 E_1 = 4 (5\ eV) = 20\ eV$$

P8. $\quad E_1 = 5 \times 10^{-18}\,J = \frac{\pi^2 \hbar^2}{2 m_e a^2}$

$$\Rightarrow \quad a = \pi \hbar \cdot \sqrt{\frac{1}{2 m_e E_1}} = \frac{\pi \cdot 1.05 \times 10^{-34}\,J \cdot s}{\sqrt{\frac{1}{2 (9.11 \times 10^{-31}\,kg)(5 \times 10^{-18}\,J)}}}$$

$$\approx 0.11\ nm$$