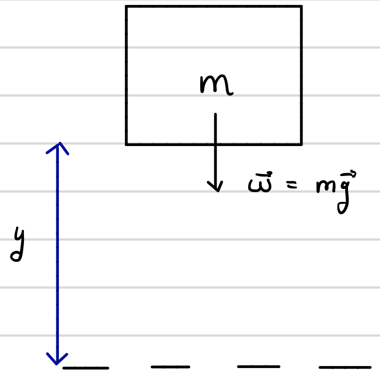
Potential Energy and Energy Conservation

Basic Equations and Concepts!

(I) Gravitational Potential Energy (U_{grav}):

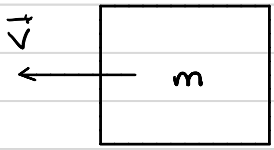


← "zero" level

$$\boxed{U_{grav} = mgy}$$

Notes: y is positive if m is above the "zero" level and negative if it's below.

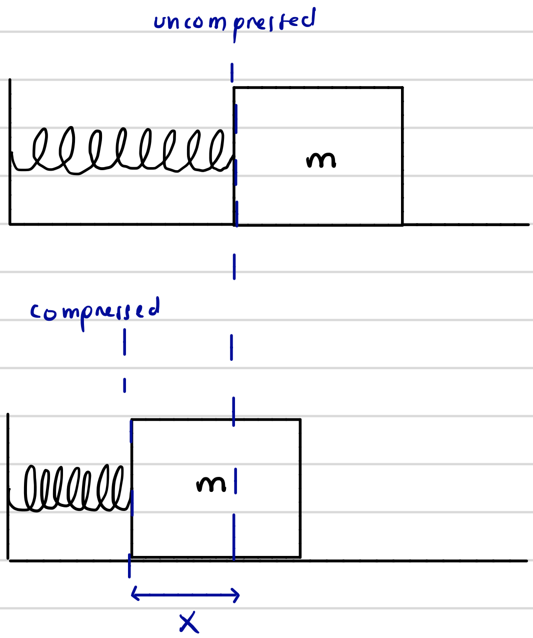
(II) Translational Kinetic Energy (K):



$$\boxed{K = \frac{1}{2} m v^2}$$

magnitude of $\vec{v}$!

(III) Elastic Potential Energy (U_{el}):

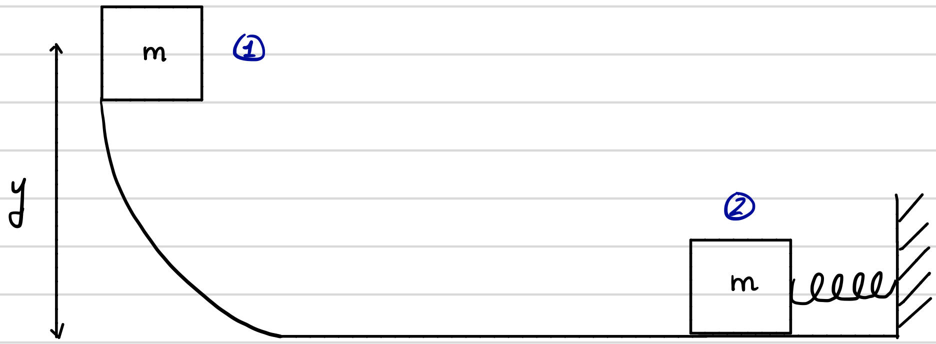


$$\boxed{U_{el} = \frac{1}{2} k x^2}$$

spring constant

Notes: the formula is the same for a spring streched a distance x.

(IV)



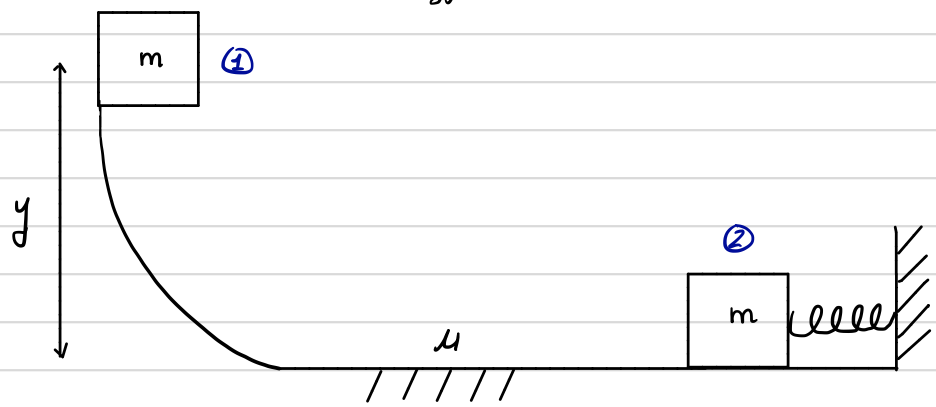
Conservation of Energy :

$$\boxed{K_1 + U_{grav,1} + U_{el\,1} = K_2 + U_{grav\,2} + U_{el\,2}}$$

$$\boxed{\frac{1}{2} m v_1^2 + m g y_1 + \frac{1}{2} k x_1^2 = \frac{1}{2} m v_2^2 + m g y_2 + \frac{1}{2} k x_2^2}$$

(V)

Conservation of Energy with other forces involved:



$$\boxed{E_1 + W_{other} = E_2}$$

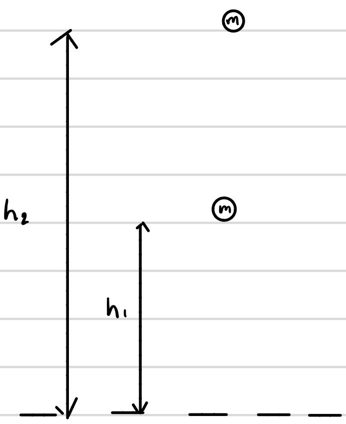
$$\boxed{K_1 + U_{grav,1} + U_{el\,1} + W_{other} = K_2 + U_{grav\,2} + U_{el\,2}}$$

Problem Solving

I. (P. 7.1)

Day 1.

Knowns: $h_1 = 1,500$ m
$h_2 = 2,400$ m
$m = 75$ kg



Objective: $\Delta U = ?$

Solution:

$$\Delta U = U_f - U_i = mgh_2 - mgh_1$$

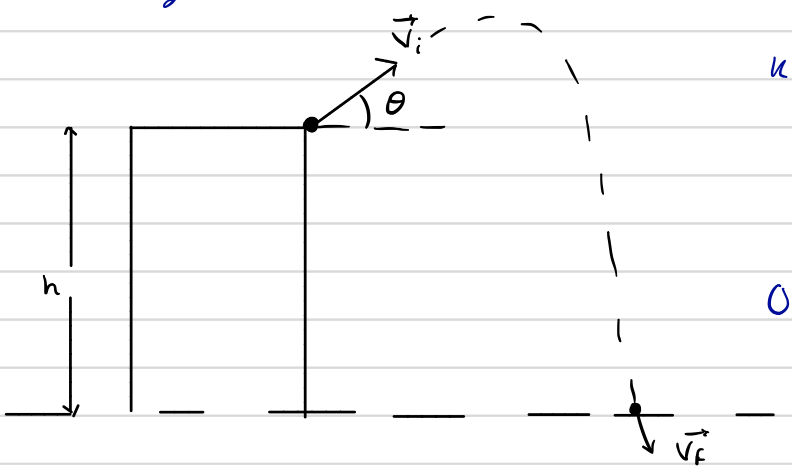
$$\boxed{= mg(h_2 - h_1)}$$

Evaluation: $\Delta U = (75\ kg)(9.8\ m/s^2)(2,400\ m - 1,500\ m)$

$\approx$

II. (P. 7.5)

Drawing:



Knowns: $y_i = h = 22.0$ m ; $y_f = 0$
$v_i = 12.0$ m/s
$\theta = 53.1^\circ$

Objective: $v_f = ?$

Zero level

Solution: Use conservation of energy.

$$\frac{1}{2}\cancel{m}v_i^2 + \cancel{m}g\,y_i = \frac{1}{2}\cancel{m}v_f^2 + \cancel{mg\,y_f}$$

$$\Rightarrow \quad \frac{1}{2} v_i^2 + g h = \frac{1}{2} v_f^2$$

$$\Rightarrow \quad \boxed{\sqrt{v_i^2 + 2 g h} = v_f}$$

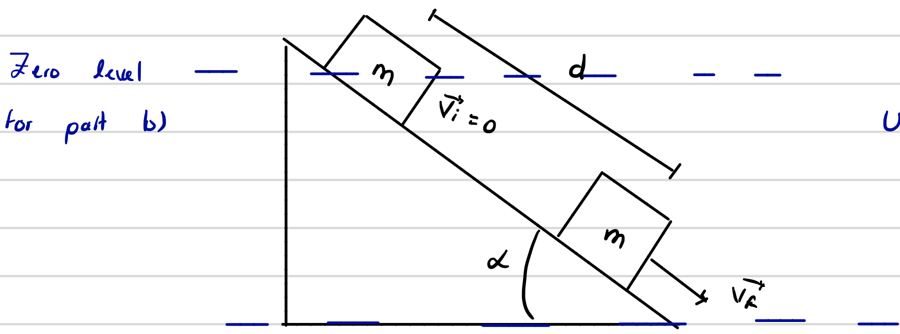
Evaluation: $\quad v_f = \sqrt{(12.0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(22.0 \text{ m})}$

$$\approx$$

Question: would this change for part b)? Nah. We only care about the magnitude of v_i.

III. (P. 7.6)

Drawing:



Zero level for part a)

Knowns: $v_i = 0$, d, m, α

Unknowns: $v_f = ?$

Solution: (Part a)

$$\frac{1}{2} m \cancel{v_i^2}^{\,0} + \cancel{mg}\, y_i = \frac{1}{2} \cancel{m}\, v_f^2 + \cancel{mg}\, \cancel{y_f}^{\,0} \quad ; \quad y_i = d \sin \alpha$$

$$\Rightarrow \quad g\, d \sin \alpha = \frac{1}{2} v_f^2$$

$$\Rightarrow \quad \boxed{v_f = \sqrt{2 g d \sin \alpha}}$$

(Part b)

$$\frac{1}{2} m \cancel{v_i^2}^{\,0} + mg \cancel{y_i}^{\,0} = \frac{1}{2} \cancel{m} v_f^2 + \cancel{m} g\, y_f$$

and $y_f = -d \sin \alpha$ (negative because it's below the zero level!)

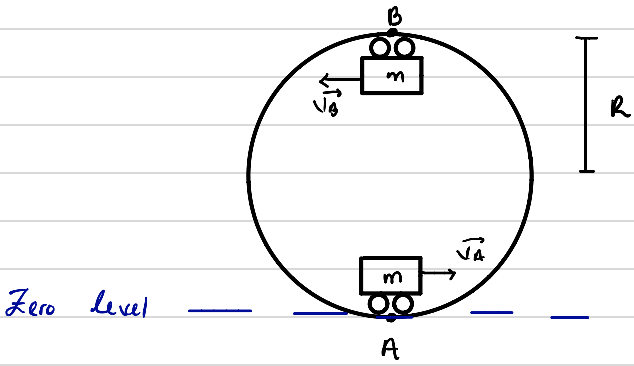
$$\Rightarrow \quad 0 = \frac{1}{2} v_f^2 - g d \sin \alpha$$

$$\Rightarrow \quad \boxed{v_f = \sqrt{2 g d \sin \alpha}}$$

Same answer b.c. choice of zero level does not matter.

IV. (P. 7.11)

Drawing



Knowns: $m = 120$ kg
$R = 12.0$ m
$v_A = 25.0$ m/s
$v_B = 8.0$ m/s

Unknowns: $W_{fr} = ?$

Solution:

$$K_A + \cancel{U_A}^{\,0} + W_{fr} = K_B + U_B$$

$$\Rightarrow \quad \frac{1}{2} m v_A^2 + W_{fr} = \frac{1}{2} m v_B^2 + mg\,(2R)$$

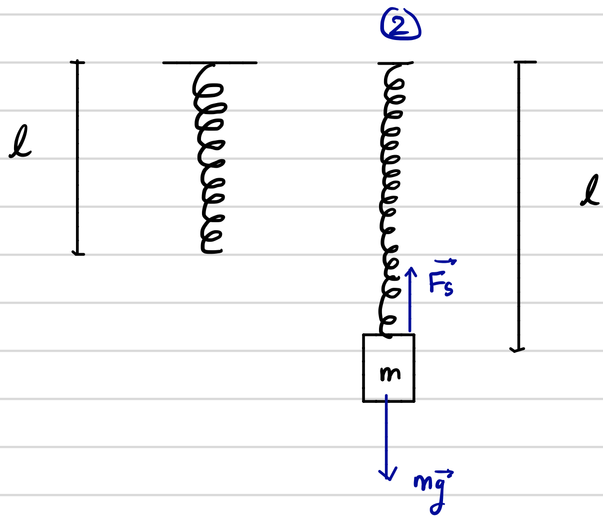
$$\Rightarrow \quad \boxed{W_{fr} = \frac{1}{2} m \left(v_B^2 - v_A^2 \right) + mg\,(2R)}$$

Evaluation:

$$W_{fr} = \frac{1}{2} (120\ \text{kg}) \left((8.0\ \text{m/s})^2 - (25.0\ \text{m/s})^2 \right) + (120\ \text{kg})(9.8\ \text{m/s}^2)(2 \cdot 12.0\ \text{m})$$

$$\approx$$

V. (P. 7.14)



Knowns: $\ell = 12.00$ cm

$m = 3.15$ kg

$\ell + x = 13.40$ cm

$U = 10.0$ J

Unknowns: $x_{new} = ?$

Solution:

$$\frac{1}{2} k x_{new}^2 = U \Rightarrow x_{new} = \sqrt{\frac{2U}{k}}$$

But we don't know k! So we use figure (2):

$$\sum F_y = 0 \Rightarrow mg - F_s = 0$$

$$\Rightarrow mg = k \Delta x \Rightarrow mg = k x$$

where $x = 13.40 \text{ cm} - \ell$

$$\Rightarrow k = \frac{mg}{x}$$

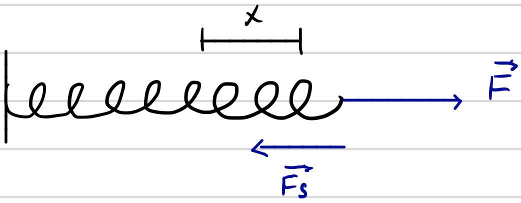
$$\therefore \quad x_{new} = \sqrt{\frac{2U}{mg} x}$$

Evaluation: $x_{new} = \sqrt{\frac{2(10.0\,J)}{(3.15\,kg)(9.8\,m/s^2)} (13.4 \times 10^{-2} m - 12.0 \times 10^{-2} m)}$

$\approx$

VI. (P. 7.15)

Drawing:



Knowns: $F = 800$ N

$x = 0.200$ m

Unknowns: $U = ?$

Solution: (Part A)

$$U = \frac{1}{2} k x_A^2 \qquad (x_A = 0.200 \text{ m})$$

Again we don't know k! But we know,

$$\sum F_x = F - F_s = 0$$

$$\Rightarrow F = F_s = kx$$

$$\Rightarrow k = \frac{F}{x}$$

$$\therefore \boxed{U = \frac{1}{2} \frac{F}{x} x_A^2}$$

Evaluation: $U = \frac{1}{2} \frac{(800 \text{ N})}{(0.200 \text{ m})} (0.200 \text{ m})^2 \approx$

(Part B) Everything is similar, we just change $x_A \mapsto x_B$

$$\boxed{U = \frac{1}{2} \frac{F}{x} x_B^2}$$

Evaluation: $U = \frac{1}{2} \frac{(800 \text{ N})}{(0.200 \text{ m})} (5.00 \times 10^{-2} \text{ m})^2 \approx$

VII. (P. 7.19)

Knowns: $k = 1600 \ N/m$

$U = 3.20 \ J$

* I'm going to skip this drawing.

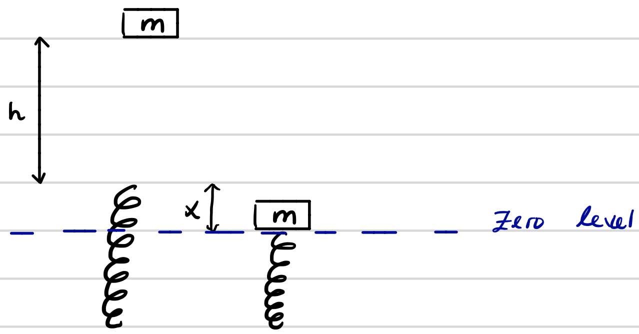
Unknowns: $x = ?$

Solution: (Part A)

$$U = \frac{1}{2} k x^2 \Rightarrow \boxed{x = \sqrt{\frac{2U}{k}}}$$

Evaluation: $x = \sqrt{\frac{2(3.20 \ J)}{(1600 \ N/m)}} \approx$

(Part B) Solution:



$$mg(h + x) = \frac{1}{2} k x^2 \Rightarrow 0 = \frac{1}{2} k x^2 - mg \, x - mgh$$

$$\boxed{\therefore \ x = \frac{mg \pm \sqrt{(-mg)^2 + 2kmgh}}{k}}$$

Evaluation:

$$x_{max} = \frac{(1.20 \ kg)(9.8 \ m/s^2) \pm \sqrt{[(1.20 \ kg)(9.8 \ m/s^2)]^2 + 2(1600 \ N/m)(1.2 \ kg)(9.8 \ m/s^2)(0.8 \ m)}}{(1600 \ N/m)}$$

$\approx$

VIII. (P. 7.20)

Solution : $\frac{1}{2} kx^2 = mgh \Rightarrow \boxed{\frac{kx^2}{2mg} = h}$

Evaluation :

$$h = \frac{(1800 \ N/m)\ (15 \times 10^{-2}\ m)^2}{2(1.20\ kg)\ (9.8\ m/s^2)} \approx$$

IX. (P. 7.23)

Solution : (Part A)

$$U = \frac{1}{2} mv^2 \Rightarrow \boxed{v = \sqrt{\frac{2U}{m}}}$$

Evaluation : $v = \sqrt{\frac{2\ (11.5\ J)}{2.5\ kg}} \approx$

Solution: (Part B)

$$\boxed{a_{max} = \frac{F_{max}}{m} = \frac{k\ x_{max}}{m}}$$

Evaluation :

$$a_{max} = \frac{(2500\ N/m)\ \sqrt{2(11.5\ J)\ /(2500\ J/m)}}{2.5\ kg}$$

$$\frac{1}{2} k x_{max}^2 = U \Rightarrow x_{max} = \sqrt{\frac{2U}{k}}$$